# Generation of arrays of elliptical holes

The first effect considered is the role of eccentricity. We investigated it by producing a regular matrix of centers of hexagonal cells forming a honeycomb tessellation, formed by elliptic holes with major semi-axis randomly oriented with respect to a common direction. A typical configuration is shown in (**Figure S1**). The geometric domain is squared of side 1. For this set of simulations, $n\_{1}=30$ nodes are placed in the horizontal direction, see (**Figure S1**). Let $h$ be the semi-distance between two centers; from the geometry of equilateral hexagons (**Figure S2**), it follows that $\frac{L}{n\_{1}}=2h=\sqrt{3}δ$ since $δ=\frac{\sqrt{3}}{2}h$. The maximum number $n\_{2}$ of rows along the vertical direction is dictated by the relation $\frac{L}{n\_{2}}=δ+\frac{δ}{2}=\frac{3L}{2\sqrt{3}n\_{1}}$, and it follows that $n\_{2}=\left⌊\frac{2\sqrt{3}n\_{1}}{3}\right⌋$, where $\left⌊\right⌋$ is the floor operator. For $n\_{1}=30$, the honeycomb geometry dictates $n\_{2}=34$ rows along the vertical direction. In order to simulate the effect of the eccentricity $e$, discrete values of this parameters have been considered ranging from 0 to 0.9 with step 0.1. The geometry is completed with the fixed elliptic hole semi-axis $a=\frac{h}{2}=\frac{L}{2n\_{1}}$, whereas the minor semi-axis varies as a function of the eccentricity as $b=a\sqrt{1-e^{2}}$.

A new grid is generated from the randomly oriented eccentric holes by considering new center to center distances obtained by considering the segments that join two centers, and the corresponding length of the segment between the two elliptic boundaries as illustrated in (**Figure S3**) for a typical elliptic hole. For every value of the eccentricity and for every trial, we then build a distribution of center to center distances that is used to generate fictitious grids of circular holes of radius $a$ with variable center-to-center distance sampled from the distribution. A typical example is given in (**Figure S4**).

# Generation of arrays of circular holes with variable diameter

The effect of variations in the holes’ diameter is simulated by considering the same honeycomb tessellation as above, but instead of assigning randomly oriented elliptic shapes with fixed major semi-axis, here we consider circular holes with randomly variated diameters within 5% to 90% (5% step) of a base diameter of size $a=\frac{h}{2}$, where $h$ is defined above. A typical configuration is shown in (**Figure S5**). As in the case of variation of eccentricity, simulation data was generated for three trials of randomly assigned diameters sampled within the above specified variations with respect to the common size $a$, and distributions of center to center distances were obtained for each discrete diameter variation. As result, fictitious distributions of nodes were generated, where the effect of diameters’ variation is quantified by considering holes of uniform diameter $a$ with variable center-to-center distance sampled from the distribution obtained from the original holes with variable diameters. An example with 60% diameter variation is shown in (**Figure** **S6**), where larger distortions appear moving away from the boundaries originating from (0,0) due to the way the center-to-center distortion is propagated.

# Combined effect of eccentricity and hole diameter

The effects of the random eccentricity and the random variation of the holes’ diameter are combined into a set of simulations based on the data generated for simulations that illustrate the effects in isolation. Therefore, we consider three trials which combine the homologous random orientation data for eccentricities $e=0.4, 0.5, 0.6$ with random diameter variation data for 10%, 15%, and 20% variations, resulting into a total of 9 combinations for each trial. A representative configuration is shown in (**Figure S7**). The procedure to calculate center to center distance is repeated, with the generation of a fictitious holes’ distribution with constant diameter $a$ and center-to-center distance sampled from the generated distribution.

# Effect of clustering

The HC substrate generated by a 2-step anodization process is characterized by smaller nanotubes (s-HC) clustered within larger domains (L-HC). This morphology was replicated in our simulations is shown in (**Figure S8**), resulting in clusters with centers in blue and nanotube indenting boundaries represented by circles.

## Rigid clusters with inter nodal stretching

This set of simulations is generated by considering fixed holes within each cluster and varying center-to-center distance. Due to the honeycomb pattern, the undeformed center to center distance in (**Figure S8**) is $6h$; which is incremented according to the discrete steps $Δ=\left\{0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2}\right\}/d$, where $d=4.5a$ is the clusters’ diameter.

## Fixed inter nodal distances with deformed clusters

By considering the same base pattern in (**Figure S8**), this set of simulations is generated by keeping the clusters’ centers fixed with respect to each other, and by radially changing within each cluster the distance from the center of the surrounding nodes. The distance is changed in five discrete steps according to $a\left(Δ\_{i}+1\right)$, with $Δ\_{i}=\frac{0.75}{4}i$, $i=0,1,…,4$. Here, $a$ is the holes’ diameter and $4.5a$ is the clusters’ diameter. Therefore, the cluster’s radius is $2.75a$, and the space not occupied by holes along the radial direction is $2.75a-\left(a+0.5a\right)=0.75a$, which explains the choice of the scaling factor in the expression for $Δ\_{i}$.

# Figures

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**Figure S1**



**Figure S2**



**Figure S3.**



**Figure S4**



**Figure S5**



**Figure** **S6**



**Figure S7**

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**Figure S8**