Supplement

Bramwell–Hill equation is represented by Seq.1.

$$PWV^2 = \frac{dP}{dD} \times \frac{D}{2\rho} \qquad \text{Seq.1}$$

(PWV: pulse wave velocity, P: blood pressure, D: blood vessel diameter, ρ: blood density)

Here, $\frac{dP}{dD} \times \frac{D}{2}$ is the bulk modulus, which represents how hard it is for the vessel to expand in diameter D against the change of pressure P. Seq.1 is a general equation expressing the relationship between the wave velocity and the bulk modulus. If the bulk modulus is constant, the wave velocity PWV is also constant. In actual blood vessels, P changes exponentially with the change of D, which is indicated by the stiffness parameter β equation of Seq.2

$$P = P_0 \times e^{\beta \times (\frac{D}{D_0} - 1)} \qquad \text{Seq.2}$$

(β: specific stiffness of the blood vessel, P_0: reference pressure, D_0: blood vessel diameter at P_0)

This exponential nature of the blood vessel causes blood pressure dependency in PWV as follows:

When both sides of Seq.2 are differentiated with D, the exponent $\frac{\beta}{D_0}$ becomes a coefficient from the differential formula of exponential,

$$\frac{dP}{dD} = \frac{\beta}{D_0} \times P \qquad \text{Seq.3}$$

Substituting Seq.3 for the right side of Seq.1,

$$PWV^2 = \frac{\beta}{D_0} \times P \times \frac{D}{2\rho} \qquad \text{Seq.4}$$

β is represented by Seq.5 by transforming Seq.4.

$$\beta = \frac{2\rho \times PWV^2}{P} \times \left(\frac{D_0}{D}\right) \qquad \text{Seq.5}$$

By dividing both sides of Seq.2 by P_0 and taking the natural logarithm,

$$\ln\left(\frac{P}{P_0}\right) = \beta \times \left(\frac{D - D_0}{D_0}\right) \qquad \text{Seq.6}$$

From Seq.6, $\frac{D0}{D} = 1/\left(1 + \frac{\ln\left(\frac{P}{P0}\right)}{\beta}\right)$ Seq.7

Substitute Seq.7 for Seq.5 and organize it with β,

$$\beta = \frac{2\rho \times PWV^2}{P} - \ln\left(\frac{P}{P_0}\right) \qquad \text{Seq.8}$$

Since in the physiological range, the second term of right side of Seq.8 is generally small compared to the first term, PWV^2 is approximately proportional to β and P.